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A Mini-Review of Constraints on Extra Dimensions

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Abstract

We present a mini-review of present constraints of the large extra dimension scenario. We show many experiments and considerations that can constrain the fundamental scale of the large extra dimension. We observe that constraints come from collider experiments are much weaker than those of astrophysical and cosmological considerations. When the number of extra dimension n is smaller than 4, the constraint is so strong that the large extra dimension scenario cannot solve hierarchy problem. But when $n \geq 4$ there is still possibility that it can solve the hierarchy problem.

1 Introduction

It was believed that because of the large Planck scale: $G_N^{-1/2} = 2.4 \times 10^{18}$GeV, the gravity is so weak that the particle physics cannot explore the world of gravity. The hierarchy between the weak scale $G_F^{-1/2} \sim 100$GeV and the Planck scale is the most serious problem in the Standard Model (hierarchy problem). Conventionally, it was explained by supersymmetry. The successful supersymmetric $SU(5)$ gauge coupling unification also encourage the existence of low-energy supersymmetry.

But no supersymmetric particles are found so far, and thus it is worthwhile to search for alternatives. In fact, Arkani-Hamed, Dimopoulos and Dvali (ADD) proposed [1] that the weakness of the gravity can be explained by the existence of extra dimensions. We are trapped in '3-brane' [2] of the higher-dimensional spacetime, and only graviton can propagate the compactified extra dimensions. Thus the overlapping between the Standard Model particles and gravitons becomes small, and the weakness of gravity can be explained. The true fundamental scale M_D is the weak scale itself, and no hierarchy problem exists. It is related to the volume of extra dimension V_n and the appearance fundamental scale $M_{pl} = 2.4 \times 10^{18}$ by

$$M_D^2 \sim V_n M_{pl}^{n+2}, \tag{1}$$

where n denotes the number of extra dimensions. We assume $V_n \sim R^n$, where R is the radius of the compactified extra dimensions. If $n = 1$, R is so large that the successful Newtonian gravity is modified, and this scenario is excluded. But if $n \geq 2$, $R \leq 1$mm. In this region, the Newtonian gravity is not tested, and thus this scenario may survive.

Also, Randall and Sundrum proposed that warped spacetime can lower the fundamental scale [3]. But we do not consider their scenario in this mini-review, and concentrate on the proposal of ADD. This scenario is called as the large extra dimension.

The very low fundamental scale $M_D \sim 1$TeV naively becomes a source of very rapid proton decay, unacceptable FCNC, $K - \bar{K}$, $B - \bar{B}$ oscillations and rare decays like $\mu \to e\gamma$. But there are many proposals against them. Therefore it is worthwhile to consider the ADD scenario in detail.

This mini-review is aimed to summarize the currently obtained constraints from many experiments. This mini-review is organized as follows: in section 2, we show the already obtained constraints from collider experiments. in section 3, we consider the astrophysical and cosmological constraints on the ADD scenario. And in section 4 we summarize.

2 Constraints from Collider Experiments

Up to now, LEP, Tevatron and HELA give lower bound on the fundamental scale M_D. We discuss the detail of their result in this section. There is another review about this issue [4].

2.1 Real Graviton Emission

Gravitons only weekly interact with other particles. Their interactions are suppressed by $1/M_{pl}^2$. But the huge number of Kaluza-Klein graviton modes enable us to observe graviton emission process, like $e^+e^- \to \gamma/Z + G$. The graviton Kaluza-Klein modes have masses equal to $|n|/R$ and therefore the different excitations have the mass splittings [5] :

$$\Delta m \sim \frac{1}{R} \sim \left(\frac{M_D}{\mathrm{TeV}}\right)^{\frac{n+2}{2}} 10^{\frac{12n-31}{n}} \ \mathrm{eV}. \tag{2}$$

The enormous number of Kaluza-Klain modes enable us to detect the massive graviton emission processes. The number of modes with Kaluza- Klein index between $|n|$ and $|n| + dn$ is :

$$dN = S_{n-1}|n|^{n-1}dn, \quad S_{n-1} = \frac{2\pi^{n/2}}{\Gamma(n/2)}, \tag{3}$$

where S_{n-1} is the surface of a unit radius sphere in n dimensions. From Eq. (1) and $m = |n|/R$, we obtain

$$dN = S_{n-1}\frac{M_{pl}^2}{M_D^{2+n}} m^{n-1} dm. \tag{4}$$

This numerator factor M_{pl}^2 is the resultant of the enourmous number of Kaluza-Klein modes, and it completely cancels the suppression factor of real graviton emission, $1/M_{pl}^2$.

L3 [6, 7] and H1 [8] searched for this process. [7] set the stringent bound on the fundamental scale. It is shown in table 1.

n	2	3	4	5	6	7
M_D (GeV)	1018	812	674	577	506	453

Table 1: The obtained lower bound on the fundamental scale M_D from real graviton emission.

2.2 Virtual Graviton Exchange

Now we want to study the virtual graviton exchange processes. We concentrate on s-channel processes, but t- and u- channel exchange processes are completely analogous. The scattering amplitude is [5]:

$$A = \mathcal{S}(s)\mathcal{T}, \tag{5a}$$

$$\mathcal{S}(s) \equiv \frac{1}{M_{pl}^2} \sum_n \frac{1}{s - m_n^2}, \tag{5b}$$

$$\mathcal{T} \equiv T_{\mu\nu}T^{\mu\nu} - \frac{1}{n+2} T_\mu^\mu T_\nu^\nu. \tag{5c}$$

Again, this amplitude is suppressed by M_{pl}. But enormous number of Kaluza-Klein modes enables us to investigate the virtual graviton exchange processes by $e^+e^- \to \gamma\gamma$, $e^+e^- \to \mu^+\mu^-$, etc.

ALEPH [9], DELPHI [10], L3 [6], OPAL [11], H1 [8] and D0 [12] searched for this virtual graviton exchange processes. Among them, the constraint from D0 collaboration is the most stringent. It is shown in table 2.

n	2	3	4	5	6	7
M_D (TeV)	1.4	1.4	1.2	1.1	1.0	1.0

Table 2: The obtained lower bound on the fundamental scale M_D from virtual graviton exchange.

3 Constraints from Astrophysical and Cosmological Considerations

If we consider the large extra dimension scenario, there are Kaluza-Klein graviton modes in the universe. They affect the evolution of supernovas, neutron stars, cosmic diffuse gamma rays, and lead to early matter domination. And furthermore, the ultra-high energy cosmic rays can create black holes and may be detected by cosmic ray detectors. From these considerations, we can set very strong bound on the fundamental scale M_D.

3.1 Supernova and Neutron Star

The existence of massive Kaluza-Klein gravitons affects the phenomenology of SN1987A and neutron star.

SN1987A lose energy by Kaluza-Klein graviton emission. such gravitons are created by $\gamma\gamma \to GG,\ e^+e^- \to GG,\ e\gamma \to eG$, gravi-bremsstrahlung and nucleon-nucleon bremsstrahlung. Therefore in order not to lose too much energy by graviton emission, the fundamental scale should not be large. It is [13]:

$$M_D \geq 30 - 130 \text{ TeV } (n = 2), \tag{6a}$$

$$M_D \geq 2.1 - 9.3 \text{ TeV } (n = 3). \tag{6b}$$

The obtained bounds depend on the supernova temperature.

Next, we consider neutron stars. The Kaluza-Klein gravitons around neutron stars decay into photons, electrons, positrons, and neutrinos. They hit the neutron stars and heat them. The requirement that neutron stars are not excessively heated by Kaluza-Klain graviton decay products implies [14] :

$$M_D \geq 1700 \text{ TeV } (n = 2), \tag{7a}$$

$$M_D \geq 60 \text{ TeV } (n = 3). \tag{7b}$$

3.2 Cosmic Diffuse Gamma Ray

In standard cosmology, big bang nucleosynthesis (BBN) provides a detailed and accurate understanding of the observed light element abundances. But the existence of massive Kaluza-Klein gravitons can destroy BBN completely. we must set "normalcy temperature" T_*, when the extra dimension are virtually empty of energy density. T_* is about 1MeV, and the result highly depend on the value of T_*.

After successful BBN, Kaluza-Klein gravitons are produced through the process $\nu\bar{\nu} \to G$, for example. And when it decay into two photons, it can destroy the observed cosmic diffuse gamma ray background. It have been measured in the 800keV to 30MeV energy range, and from the experiment we can set [15] :

$$M_D \geq 110 - 350 \text{ TeV } (n = 2), \tag{8a}$$

$$M_D \geq 5.0 - 13.8 \text{ TeV } (n = 3). \tag{8b}$$

3.3 Early Matter Domination

Again, Kaluza-Klein massive gravitons are created by gravi-bremsstrahlung etc. Such massive gravitons inject extra massive matter in the universe, and lead to a more rapid decline in the CMB temperature. The increased cooling rate means that by the time the CMB has cooled to 2.73K the universe is still be much too young to hold the objects we observe in ours. From this fact, we can set lower bound on fundamental scale. The energy density of massive gravitons highly depends on the QCD scale, and so the obtained bound also depend on it. The bound is [16] :

$$M_D \geq 86 - 1000 \text{ TeV } (n = 2), \tag{9a}$$

$$M_D \geq 7.4 - 59 \text{ TeV } (n = 3), \tag{9b}$$

$$M_D \geq 1.5 - 9.0 \text{ TeV } (n = 4). \tag{9c}$$

3.4 Black Hole Production by Cosmic Rays

If the large extra dimension scenario is true, black holes are created when the collision energy of two particles is larger than the fundamental scale. Ultra-high energy cosmic rays provide the most promising window to observe black holes before LHC starts [17]. Cosmic rays penetrate the atmosphere of the earth and collide with some nuclei, producing black holes. They immediately evaporate and lead to energetic showers. AGASA already provides the most stringent bound on the fundamental scale for $n \geq 5$. It is [18] :

$$M_D \geq 1.3 - 1.5 \text{ TeV } (n = 4), \tag{10a}$$

$$M_D \geq 1.4 - 1.6 \text{ TeV } (n = 5), \tag{10b}$$

$$M_D \geq 1.5 - 1.7 \text{ TeV } (n = 6), \tag{10c}$$

$$M_D \geq 1.6 - 1.8 \text{ TeV } (n = 7). \tag{10d}$$

4 Summary

To summarize, we have shown the currently obtained lower bound on the fundamental scale of the large extra dimension. It is summarized in table 3. You notice that the constraints from present colliders are so weak that astrophysical and cosmological ones dominate. If $n \leq 3$, the constraints from astrophysics and cosmology are so stringent that we cannot solve the hierarchy problem. But if $n \geq 4$, the constraints are very weak and there is still possibility that the large extra dimension is the solution of the hierarchy problem.

n	2	3	4
M_D (TeV)	1700 (neutron star)	60 (neutron star)	1.5 (early matter domination)
n	5	6	7
M_D (TeV)	1.3 (AGASA)	1.4 (AGASA)	1.5 (AGASA)

Table 3: The most conservative bound on the fundamental scale of the large extra dimension.

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